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ALGORITHMS FOR CALCULATING AND CORRECTING BLOOD-GAS AND ACID-BASE VARIABLES¹

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ABSTRACT

This is a review of equations available for correcting Po, Pco, and pH for temperature and for estimating blood-gas and acid-base variables from measured values:

- Oxygen saturation (S_{O2}) from P_{O2}, P_{CO2}, pH, and body temperature
- oxygen concentration from S_{02} , P_{02} , and hemoglobin concentration
- Po2 at half-saturation (P50) from Po2, So2, pH, Pco2, and T;
- 4. "in vivo P₅₀" from P_{O2}, S_{O2}, and T;

 base excess (BE) from pH, P_{CO2}, and Hb;

- 6. "in vivo BE" (BE₃) from pH and P_{CO₂};

 7. "compensated BE₃" from P_{CO₂}; and

 8. bicarbonate and carbon dioxide concentrations from pH and P_{CO₂}. Physiologic considerations are emphasized, with mathematical background when it contributes to understanding. Algorithms are variously compared with their graphic counterparts, with the data from which they were derived, and with each

INTRODUCTION

With increasing availability of computers and programmable calculators, many computations previously performed in the physiology or blood-gas laboratory with analog devices, such as slide rules (Severinghaus, 1966), nomograms (Siggaard Andersen, 1963; Kelman and Nunn, 1966; Severinghaus, 1976a), and line charts (Bradley et al., 1956; Severinghaus, 1958 and 1964; Kelman and Nunn, 1966), can now be carried out more conveniently using digital techniques. To do this, one needs numeric algorithms to represent certain physical and chemical relationships in the blood:

- 1. It is sometimes useful to estimate the percentage saturation of hemoglobin with oxygen (S_{O_2}) , using measured P_{O_2} and a standard curve describing oxygen-hemoglobin equilibrium. Occasionally it is also useful to estimate P_{O_2} from S_{O_2} . Not only are accurate numeric representations of the oxygen hemoglobin equilibrium curve (OHEC) needed for these transformations, but also methods to account for shifts in position of the OHEC caused by temperature, pH, and P_{CO_2} .
- 2. Blood gases and pH are usually measured with electrodes thermostatically controlled at 37°C. Results should be corrected to the patient's body temperature in order to describe the true physiologic condition.
- 3. In assessing acid-base status from blood pH and P_{CO2}, most clinicians and physiologists find it useful to have a number representing the metabolic component of acid-base derangement. Base excess (BE) or plasma bicarbonate concentration, [HCO3], is often derived for this purpose.

This paper is to present algorithms for digitally carrying out the above corrections and transformations. Also included are evaluations of how faithfully the algorithms represent their graphic counterparts or the data from which they were derived. All of the equations are sufficiently simple that calculations can be carried out on small "hand-held" programmable calculators, such as the Hewlett-Packard HP-67 or the Texas Instruments TI-59. Alternatively, appropriate algorithms can be consolidated into a comprehensive computor program for calculating or correcting large numbers of blood-gas and acid-base variables in a busy laboratory.

METHODS AND RESULTS

I. OXYGEN HEMOGLOBIN EQUILIBRIUM CURVE (OHEC)

A. The Standard OHEC

The sigmoid curve describing relationships between P_{O_2} and S_{O_2} in blood is usually mathematically modeled from 38 data pairs reported by Severinghaus (1966) (Table 1). These matching values of S_{O_2} (1-99.95 percent) and P_{O_2} (1.9-700 torr) represent the OHEC for normal adult human blood at 37°C, pH = 7.40, and P_{CO_2} = 40 torr. In table 2 are nine algorithms for calculating S_{O_2} from P_{O_2} (Kelman, 1966; Thomas, 1972; Aberman et al., 1973; Roughton and Severinghaus, 1973; Lutz et al., 1975; Ruiz, 1975; Severinghaus, 1976b and 1979), and in table 3 three algorithms for calculating P_{O_2} from S_{O_2} (Lutz et al., 1975; Tien and Gabel, 1977; Severinghaus, 1979). Equations characterize the entire OHEC, except where a limited range is indicated.

Fidelity with which an algorithm represents the data from which it was derived is estimated from residual errors; that is, differences between the original data and values calculated from the equation. In table 4 is the range of residual errors resulting when each equation is compared with the Severinghaus OHEC. Also listed for each equation is the root mean square of residual errors (RMS); that is, the square root of the mean of the squares of all residual errors, a measure of the "average error" from applying the algorithm. Table 4 also contains the approximate time it takes to compute S_{0} or P_{0} with each equation, using a typical small programmable calculator.

Ruiz's algorithm for estimating S_{Q_2} from P_{Q_2} , although clearly superior to the others in accuracy (RMS = 0.07 percent saturation) may not be ideal for all applications because of its relatively long computation time (20 sec) when a small calculator is used. Equation #1 of Severinghaus retains reasonable accuracy (RMS = 0.34 percent) while offering rapid solution (1.5 sec). A compromise represented by Aberman's algorithm (RMS = 0.17 percent; 13.5 sec) may be preferred by some.

For estimating P_{O_2} from S_{O_2} , application of two of the algorithms in table 3, Lutz's low-range equation when S_{O_2} is below 20 percent and Tien's wide-range algorithm when S_{O_2} is 20-97.5 percent, gives a combined root mean square of 0.10 torr and residual errors from -0.21 to 0.18 torr. Although residual errors for both equations are low where they overlap between 20 and 40 percent saturation, Tien's algorithm is more accurate than Lutz's in this range.

B. Effects of Temperature, pH, and P_{CO2} on Position of the OHEC

When developing algorithms for dealing with changes in the affinity between oxygen and hemoglobin owing to temperature, pH, and P_{CO_2} , it is generally assumed that these variables influence position but not shape of the OHEC (Severinghaus, 1958, 1964, and 1966; Roughton, 1964; Astrup et al., 1965; Kelman, 1966; Aberman et al., 1975). For $S_{O_2} = 5-95$ percent where this assumption is valid, P_{O_2} is increased or decreased by a single multiplicative

factor for any combination of temperature, pH, and P_{CO_2} deviating from the standard 37° C, 7.40, and 40 torr. Hence, the S_{O_2} of blood having nonstandard temperature, pH, and P_{CO_2} can be approximated from the standard OHEC by first calculating, using an equation to be derived in the next three paragraphs, a virtual P_{O_2} (P_v) that would exist if temperature, pH, and P_{CO_2} were made standard (Severinghaus, 1958 and 1964; Kelman, 1966; Kelman and Nunn, 1966; Thomas, 1972; Ruiz, 1975). The value of P_v can then be substituted for P in one of the algorithms in table 2 to compute the S_{O_2} expected at the observed values of P_{O_2} , P_{CO_2} , pH, and temperature.

When S_{O_2} is held constant and the affinity between oxygen and hemoglobin is modified by an alteration in temperature or pH, ratios of respective values for P_{O_2} vary as the power (base 10) of the change in temperature or pH (Bradley et al., 1956; Astrup et al., 1965; Severinghaus, 1966; Kelman and Nunn, 1966):

$$P_{T_1}/P_{T_2} = \exp_{10}(f(T_1-T_2))$$
 (1)

$$P_{pH_1}/P_{pH_2} = \exp_{10}(g(pH_1-pH_2))$$
 (2)

where P_{T1} and P_{T2} are P_{O2}'s at different temperatures (°C), P_{pH1} and P_{pH2} are P_{O2}'s at different pH's, and f and g are constants derived empirically. Factors reported by Severinghaus (1966) are usually used for these conversions:

$$f = \Delta \log_{10} P_{O_2} / \Delta T = 0.024$$
 (3)

$$g = \Delta \log_{10} P_{O_2} / \Delta pH = -0.48 + (0.0013(BE) / \Delta pH)$$
 (4)

Equation 4 acknowledges that changing pH by altering blood [HCO $_3$] has less influence on the affinity between oxygen and hemoglobin than changing pH by altering P_{CO}_2 (Naeraa et al., 1963 and 1966; Severinghaus, 1966). Kelman (1966), in his equation for calculating virtual P_{O}_3 , used the form:

$$g' = \Delta \log_{10} P_{O_2} / \Delta pH = -0.40 - (0.06(\log_{10} 40 - \log_{10} P_{CO_2}) / \Delta pH)$$
 (4')

To produce a single multiplicative factor accounting for effects of both temperature and pH on oxygen-hemoglobin affinity, equation 1 is multiplied by equation 2. To do this, the exponents $f(T_1-T_2)$ and $g(pH_1-pH_2)$ or $g'(pH_1-pH_2)$ are added. Then standard values for temperature and pH are introduced, and values for f and g or g' from equations 3 and 4 or 4' are substituted. This gives two algorithms for calculating virtual P_{O_2} (P_v or P'_v) from measured P_{O_2} (P_m):

$$P_{v} = P_{m}(\exp_{10}(-0.024(T-37)+0.48(pH-7.4)-0.0013(BE)))$$

$$P'_{v} = P_{m}(\exp_{10}(-0.024(T-37)+0.40(pH-7.4)+$$

$$0.06(\log_{10}40-\log_{10}P_{CO_{2}})))$$
(5')

where P_v and P_v refer to virtual P_{O_2} 's calculated from combinations of equations 1, 2, and 3 with equations 4 and 4', respectively.

Values for the dependent variables P_v/P_m and P_v/P_m fall within one percent of each other for all independent variables in the ranges: pH = 7.2-7.6, P_{CO_2} = 30-80 torr, BE = -15 to +10 mEq/L, and Hb = 10-20 gm/100 ml. The ratios fall within 1.5 percent of each other when pH = 7.2-7.6, P_{CO_2} = 20-80 torr, BE = -15 to +15 mEq/L, and Hb = 10-20 gm/100 ml.

II. ANAEROBIC TEMPERATURE CORRECTIONS

With changes in temperature come alterations in both solubility of gases and dissociation constants of ionizable solvents and solutes. Therefore, P_{O_2} , P_{CO_2} , and pH of blood change as temperature is anaerobically altered. With cooling, pH is increased, P_{CO_2} and P_{O_2} decreased; with warming, the opposite is true.

A. pH

Rosenthal (1948) reported that pH of whole blood decreases 0.0147 as temperature is elevated 1° C; that is, $\Delta pH/\Delta T = -0.0147$. Adamsons and colleagues (1964) subsequently showed that $\Delta pH/\Delta T$ is not constant but is

dependent upon pH and carbon dioxide concentration (C_{CO2}) of the blood. They reported that:

$$\Delta pH/\Delta T = -0.0146-0.005(pH_{38}-7.4)+(5x10^{-5})(C_{CO_2}-20)$$
 (6)

where pH₃₈ is pH at 38°C. Severinghaus (1966) approximated this with

$$\Delta pH/\Delta T = -0.0146-0.0065(pH_{38}-7.4)+(3x10^{-5})(BE)$$
 (7)

and pointed out that the final term may be ignored, because the error in pH correction owing to abnormal BE would be only ± 0.006 in the extreme case when ΔT is $\pm 10^{\circ}$ C and BE is ± 20 mEq/L.

The correction per degree centigrade will be insignificantly different if temperature of the pH electrode is 37 rather than 38°C. Therefore, pH at T°C can be calculated from:

$$pH_T = pH_{37} - (T-37)(0.0146 + 0.0065(pH_{37} - 7.4))$$
 (8)

B. PCO2

When blood is anaerobically warmed or cooled, $C_{\rm CO_2}$ remains constant (Bradley et al., 1956; Severinghaus et al., 1956; Severinghaus, 1966), leaving changes in pH, dissociation constants, and solubility of carbon dioxide to account for temperature-dependent alterations in $P_{\rm CO_2}$. As with $P_{\rm O_2}$, temperature correction for $P_{\rm CO_2}$ follows an exponential relationship (Bradley et al., 1956; Nunn et al., 1965; Kelman and Nunn, 1966; Severinghaus, 1966):

$$P'_{T_1}/P'_{T_2} = \exp_{10}(f'(T_1-T_2))$$
 (9)

where P'T, and P'T, are PCO2's at different temperatures.

Although Bradley, Stupfel, and Severinghaus (1956) reported theoretical values for f' ranging from 0.0175 to 0.0222, depending upon pH and temperature, Nunn and colleagues (1965) experimentally found that f' = 0.019 for wide ranges of P_{CO_2} (18.6-70.7 torr) and temperature (18.0-36.5°C). Severinghaus (1966) subsequently advocated use of 0.019 for temperature correction near 37°C, with

the admonition that correction factors for lower body temperatures (less than 30°C) will be too large, especially if blood pH is also low. For body temperatures of 30-40°C, the following algorithm can be used to correct P_{CO2} for differences in temperature between the measuring electrode and the body:

$$P_{T} = F_{37} \exp_{10}(0.019(T-37))$$
 (10)

where P'_{T} is P_{CO_2} at body temperature (T^oC) and P'_{37} is P_{CO_2} measured at $37^{\circ}C$.

Bradley and colleagues (1956) calculated values for P'_{T}/P'_{37} using the following transformation of the Henderson-Hasselbalch equation:

$$P_{CO_2} = C_{CO_2}/(\alpha (\exp_{10}(pH-pK)+1))$$
 (11)

where α' is the solubility of carbon dioxide in plasma. Because C_{CO_2} remains constant during change in temperature (Severinghaus et al., 1956) and will cancel out in numerator and denominator, the ratio of values for P_{CO_2} at $T^{O}C$ and $37^{O}C$, expressed as in equation 11, is:

$$P'_{T}/P'_{37} = \alpha'_{37}(\exp_{10}(pH_{37}-pK_{37})+1)/$$

$$\alpha'_{T}(\exp_{10}(pH_{T}-pK_{T})+1)$$
(12)

With substitution of currently-accepted values for α'_{37} and pK $_{37}$ (37°C) and α'_{T} and pK $_{T}$ (T°C) at various values of pH (7.0-7.6) and temperature (35-40°C) (Severinghaus, 1964), ratios P' $_{T}$ /P' $_{37}$ arising from this equation fall within 1 percent of those calculated from equation 10. For correcting to progressively lower temperatures, ratios P' $_{T}$ /P' $_{37}$ calculated from equation 10 become substantially larger than those predicted from the Henderson-Hasselbalch equation (equation 12), especially if pH is low. For example, when correcting P $_{CO_2}$ of blood having pH = 7.0 from 37 to 30°C, measured P $_{CO_2}$ would be multiplied by 0.736 according to the algorithm (equation 10), by 0.712 according to the Henderson-Hasselbalch equation, a difference of 3.4 percent.

C. P_{O2}

Change in the P_{O_2} of blood with anaerobic cooling or warming in the midrange of the OHEC is caused primarily by an alteration in the affinity between oxygen and hemoglobin; that is, in position of the OHEC (Severinghaus, 1966). For S_{O_2} up to about 75 percent, the P_{O_2} correction for temperature is dependent on changes in position of the OHEC with temperature (equation 3) and pH (equation 4) and on changes in pH with temperature (equation 7). Multiplying equations 4 and 7 gives $\Delta \log_{10} P_{O_2} / \Delta T$ owing to change in pH with temperature:

 $\Delta \log_{10} P_{O_2} / \Delta T = 0.007 + 0.0031(7.4 - pH_{37}) - (1 \times 10^{-5})(BE)$ (13)

Adding this to equation 3 produces an algorithm accounting for alterations in P_{O_2} caused by a combination of the effects of temperature-dependent changes in pH and direct effects of temperature on hemoglobin-oxygen affinity (Severinghaus, 1966):

$$\Delta \log_{10} P_{O_2} / \Delta T = 0.031 + 0.0031(7.4 - \mu_{37}) - (1 \times 10^{-5})(BE)$$
 (14)

The second and third terms of this equation, having to do with dependence of $\Delta pH/\Delta T$ on pH and BE, may be ignored, since they would produce a maximum change in $\Delta \log_{10} P_{O_2}/\Delta T$ of only ± 0.0018 (± 0.4 percent error in the ratio of P_{O_2} 's at different temperatures, P_T/P_{37} , per degree centigrade) for variations in pH of ± 0.5 and in BE of ± 20 mEq/L. Hence, for midrange of the OHEC (Severinghaus, 1966):

$$\Delta \log_{10} P_{O_2} / \Delta T = 0.031$$
 (15)

This assumes constancy not only of oxygen concentration (C_{O_2}) but also of S_{O_2} during anaerobic temperature change. Saturation greater than 75-80 percent (P_{O_2} above 40-45 torr) cannot, however, be assumed to remain constant during change in temperature. When blood is cooled, some dissolved oxygen combines with hemoglobin, because during cooling the increase in affinity

between hemoglobin and oxygen is dominant over the increase in solubility of oxygen in blood (Hedley-Whyte and Laver, 1964; Nunn et al., 1965; Severinghaus, 1966; Roughton and Severinghaus, 1973). The resulting slight elevation in S_{O_2} becomes more influential near the top of the OHEC, where small increases in S_{O_2} result in large increases in P_{O_2} . Therefore, at P_{O_2} values greater than 40-45 torr, a reduction in temperature produces proportionally less decrease in P_{O_2} than at lower P_{O_2} is (fig. 1a,b). That is, the reduction in P_{O_2} with cooling, caused by both increased solubility of oxygen and increased oxygen-hemoglobin affinity, is attenuated at higher P_{O_2} is by the augmentation of S_{O_2} induced by cooling. The same is true for warming, during which the increase in P_{O_2} is attenuated at higher P_{O_2} is owing to the transfer of small amounts of oxygen from the combined to the dissolved state.

Variable $\Delta \log_{10} P_{O_2}/\Delta T$ with increasing P_{O_2} and S_{O_2} is accounted for in the data of Severinghaus (1966) that underlie the algorithm reported by Ruiz and colleagues (1975) for calculating P_{O_2} at T^OC (P_T) from P_{O_2} measured at 37^OC (P_{37}):

$$P_{T} = P_{37}(\exp_{10}((T-37)(0.0265/((P_{37}/146)^{3}+1)+ (16)$$

$$0.0007/(0.02(P_{37}-230)^{2}+1)+0.0047)))$$

An algorithm using S_{O_2} (S) rather than P_{O_2} comes from a commercial source (HP-67/HP-97 Users' Library Solutions. Pulmonary. Hewlett-Packard, 1000 N.E. Circle Boulevard, Corvalis, OR 97330):

$$P_{T} = P_{37}(\exp_{10}((T-37)(3130-62.5(S)+0.312008(S^{2})))/$$

$$(100,000-1,993(S)+9.9313(S^{2}))))$$
(17)

The terms accounting for variable $\Delta \log_{10} P_{O_2}/\Delta T$ in equations 16 and 17 agree with each other to within 2 percent when any of the first 35 pairs of values for P_{O_2} (1.9-225 torr) and S_{O_2} (1-99.5 percent) on the standard OHEC (table 1) is

fig. la,b

substituted in them. More useful for comparison than percentage differences in $\Delta \log_{10} P_{O_2}/\Delta T$, however, are percentage differences in P_T/P_{37} , which are dependent upon T-37 and which can be calculated after exponential transformation of (T-37)($\Delta \log_{10} P_{O_2}/\Delta T$):

$$P_{T} = P_{37}(\exp_{10}((T-37)(\Delta \log_{10} P_{O_{2}}/\Delta T)))$$
 (18a)

$$P_T/P_{37} = \exp_{10}((T-37)(\Delta \log_{10} P_{O_2}/\Delta T))$$
 (18b)

When $S_{O_2} = 1-99.5$ percent, factors P_T/P_{37} computed from equations 16 and 17 differ by less than 0.1 percent per degree centigrade change in temperature.

For S_{O_2} up to 99 percent (P_{O_2} to 159 torr), values of $\Delta log_{10}P_{O_2}/\Delta T$ calculated from equations 16 and 17 agree to within 2 percent with those read from the Severinghaus graph of $\Delta log_{10}P_{O_2}/\Delta T$ plotted against S_{O_2} and P_{O_2} (Severinghaus, 1966) (fig. 1a). As above, it follows that factors P_T/P_{37} from the equations differ with those from the graph by less than 0.1 percent per degree centigrade change in temperature. Therefore, either equation 16 or 17 can be used to represent the factors recommended by Severinghaus in 1966 to correct P_{O_2} for anaerobic temperature change.

Severinghaus (1979) has recently published his own algorithm for this correction:

$$P_{T} = P_{37}(\exp_{e}((T-37)(0.013+0.058/(1+0.243(P_{37}/100)^{3.88}))))$$
 (19)

(The exponent here is to the base e, a fact that will be accounted for later, when this equation is compared with algorithms having exponents to the base 10.) This equation was derived theoretically from an expression for slope of the OHEC as a function of P_{O_2} . Correction factors P_T/P_{37} calculated from equation 19 are lower by as much as 0.36 percent per degree centigrade (at P_{O_2} = 225 torr, S_{O_2} = 99.5 percent) and higher by up to 0.28 percent per degree centigrade (P_{O_2} = 99.6 torr, P_{O_2} = 97.5 percent) than those calculated from equation 16 or 17. The

three algorithms might be considered comparable when accounting for small temperature variations.

From data of Nunn and colleagues (1965), Kelman and Nunn (1966) developed another algorithm for calculating P_{O_2} at a temperature other than that at which P_{O_2} is measured:

$$P_{T} = P_{37}(\exp_{10}((T-37)(0.0052+0.0268(1-\exp_{e}(-0.3(100-S))))))$$
 (20)

Values for $\Delta \log_{10} P_{O_2}/\Delta T = 0.0052+0.0268(1-exp_e(-0.3(100-S)))$ are as much as 27 percent lower than those resulting from applying the appropriate segments of equations 16 and 17 or from reading values from the Severinghaus graph (1966) (fig. 1a); values of $\Delta \log_{10} P_{O_2}/\Delta T$ from equation 20 are up to 29 percent lower than those from equation 19. However, the exponential transformation results in values of P_T/P_{37} from equation 20 that differ by no more than 1.3 percent per degree centigrade (T-37) with those from equation 16 or 17, and by a maximum of 1.6 percent per degree centigrade with those from equation 19. Because percentage differences between the Kelman-Nunn and the Severinghaus values for $\Delta \log_{10} P_{O_2}/\Delta T$ are maximum when $P_{O_2} = 110-130$ torr (fig. 1a), percentage differences between the respective correction factors P_T/P_{37} increase to more than 5 percent for temperature variation (T-37) greater than 4° C in this range of P_{O_2} (fig. 1c). Considerations for choosing between algorithms derived from the Severinghaus (1966) or the Nunn data (1965) are beyond the scope of this paper.

III. BASE EXCESS (BE)

fig. lc

Several algorithms for calculating BE with computers are available (Dell et al., 1967; Maas et al., 1972; Thomas, 1972). Two others can be solved using small programmable calculators. Siggaard Andersen's algorithm (1966) is the simpler of the two:

BE =
$$(1-0.0143(Hb))((HCO_3^-)-(9.5+1.63(Hb))(7.4-pH_{37})-24)$$
 (21)

where Hb is the concentration of hemoglobin (gm/100ml). Substituting a form of the Henderson-Hasselbalch equation to express [HCO₃⁻] in terms of pH (pH₃₇) and P_{CO₂} (P'₃₇), both measured at 37°C:

BE =
$$(1-0.0143(Hb))(0.0306(P_{37})(exp_{10}(pH_{37}-6.1))$$
-
$$(9.5+1.63(Hb))(7.4-pH_{37})-24)$$

Included are constant values for apparent pK (6.1) and solubility of carbon dioxide in plasma (0.0306 mM/(liter x torr)). No improvement in accuracy results from correcting pK' for pH using a linear regression of data reported by Severinghaus (1964): $pK'_{37} = 6.4104-0.04214(pH_{37})$.

Values of BE calculated with Siggaard Andersen's algorithm are generally more positive or less negative than those read from the Severinghaus Blood Gas Calculator (BGC) $(1966)^2$. For the following ranges: BE = -15 to +15 mEq/L, $P_{CO_2} = 20$ -80 torr, pH = 7.2-7.6, and Hb = 10-20 gm/100ml, equation 22 underestimates negative BE by up to 1.9 mEq/L³ and overestimates positive BE by as much as 1.3 mEq/L⁴. Percentage error is greatest at BE = 0 mEq/L, where overestimation is 1.4 mEq/L when $P_{CO_2} = 21.1$ torr, pH = 7.60, and Hb = 10 gm/100ml (table 5).

Gershwin and colleagues (1974) developed an iterative digital technique for calculating BE from the point of intersection between a mathematically-modeled blood buffer line and the base excess curve on Siggaard Andersen's acid-base nomogram (1962) (fig. 2). Slope of the buffer line is approximated by:

fig. 2

$$s = -1.16(exp_e(0.0204(Hb)-0.01434(BE)))$$
 (23a)

pH at which the buffer line intersects the base excess curve (fig. 2) is found by simultaneously solving equations representing the two lines, each expressed as a function of $\Delta \log_{10} P_{CO_2}/\Delta T$ and pH:

$$pH' = (-B-(B^2-4AC)^{0.5})/2A$$
 (23b)

where A = s+0.77

$$B = \log_{10} P_{CO_2}^{-s(pH)-7.55-7.165(s+0.77)}$$

$$C = -7.165(\log_{10} P_{CO_2}^{-}) + 7.165(s)(pH) + 7.55(7.165) + 0.058$$

To calculate BE, the value of pH' is substituted in an empiric equation for the base excess curve, expressed as a function of pH alone:

BE =
$$59.4(\log_{10}(3.36(pH'-0.1)))$$
 (23c)

In practice, an initial estimate of 0 mEq/L is substituted for BE in equation 23a. Subsequent estimates obtained from 23c are then reintroduced into 23a until equations 23a, b, and c have been sequentially applied three times. (These calculations take about 18 sec on the HP-67 programmable calculator.) The result after three iterations is considered the best estimate of BE; results from further iterations sometimes diverge from the real value.

Gershwin's algorithm probably provides the best available method for calculating BE using small programmable calculators. When BE = -15 to +10 mEq/L, P_{CO_2} = 20-80 torr, pH = 7.2-7.6, and Hb = 10-20 gm/100ml, it gives values for BE that are within ±0.8 mEq/L of values read from the BGC (table 5). For perspective, differences in BE of ±0.8 mEq/L can be produced by simultaneous variations in P_{CO_2} and pH of ±0.5 torr and ±0.005, respectively, at extremes of the above ranges.

If Hb is low and P_{CO_2} high, equations 23a, b, and c consistently overestimate values of BE above +3 mEq/L. Errors become progressively greater as BE increases and can become considerable when BE exceeds +10 mEq/L (fig. 3). When Hb = 10 gm/100ml and P_{CO_2} = 80 torr, the overestimation is 2.4 mEq/L at BE = +15 mEq/L, pH = 7.347, and is 4.1 mEq/L at BE = +20 mEq/L, pH = 7.393. For Hb = 5 gm/100ml, errors at the above P_{CO_2} and BE are +4.4 and +8.8 mEq/L,

respectively. When Hb = 5 gm/100ml and P_{CO_2} = 60 torr, the algorithm overestimates BE by 2.6 mEq/L at BE = +15 mEq/L, pH = 7.433, and by 5.4 mEq/L at BE = +20 mEq/L, pH = 7.480. Therefore, Gershwin's algorithm, although quite good otherwise, should not be used for severely anemic patients with substantial degrees of partially-compensated respiratory acidosis or metabolic alkalosis.

There are limitations to the usefulness of BE, however determined, for planning treatment of patients with metabolic acid-base derangements. A major problem is that BE is defined according to the way pH of fully-oxygenated blood changes as P_{CO2} is systematically altered in vitro (Astrup et al., 1960). Many have pointed out that in vivo, where carbon dioxide and bicarbonate are distributed in interstitial fluid as well as in blood, BE exhibits a different relationship to pH and P_{CO} than in the test tube (Schwartz and Relman, 1963; Brackett et al., 1965 and 1969; Brown and Clancy, 1965; Michel et al., 1966; van Ypersele de Strihon et al., 1966; Arbus et al., 1969). Severinghaus (1976a) proposed that the "in vivo BE" (BE3) be read from Siggaard Andersen's alignment nomogram at 3 gm/100ml, an "average" hemoglobin concentration in the extracellular fluid. Hence, when bicarbonate and carbon dioxide re-equilibrate and redistribute as P_{CO2}, in the blood is altered, the BE₃ remains relatively constant, signifying constancy of the metabolic acid-base state. The conventional BE would change under these circumstances, deceptively implying an increase or decrease in nonvolatile acid.

Severinghaus' algorithm (1976a) for approximating BE2 is:

$$BE_{3} = 37(\exp_{e}((pH-7.4+0.345(\log_{e}(P_{CO_{2}}/40))))/$$

$$(0.55-0.09(\log_{e}(P_{CO_{2}}/40))))-1)$$
(24)

For BE = -20 to +15 mEq/L and P_{CO_2} = 10-80 torr, BE₃ calculated with this equation is within ±0.3 mEq/L of that read from a special modification of Siggaard Andersen's alignment nomogram (Severinghaus, 1976a) (table 5). Differences between the algorithm and the nomogram are less than 1 mEq/L for BE = -30 to +25 mEq/L, P_{CO_2} = 10-80 torr.

Severinghaus (1976a) also reported an equation for calculating the "compensated BE3" (BE_C) that would be expected, according to the data of Brackett and colleagues (1969), after full renal compensation for prolonged hypercapnia:

$$BE_{c} = 15(\log_{e}(P_{CO_{2}}/40)) + 0.095(P_{CO_{2}}-40)$$
 (25)

This equation, too, matches the nomogram to within ±0.3 mEq/L for P_{CO2} up to 80 torr. Dividing equation 24 by equation 25 gives an indication of the degree of metabolic compensation for respiratory acidosis.

For those who find the plasma bicarbonate concentration helpful when clinically managing acid-base disorders, the following version of the Henderson-Hasselbalch equation can be used:

$$[HCO_3^-] = \alpha'(P_{CO_2})(exp_{10}(pH-pK'))$$
 (26)

where pK' and α' (solubility of carbon dioxide in blood plasma) are appropriate for temperature and pH. For 37°C, the following includes a value for α' and an estimate of pK' corrected for pH, according to data reported by Severinghaus (1964):

$$[HCO_3^-] = 0.0306(P_{CO_2})(exp_{10}(1.04214(pH)-6.4104))$$
 (27)

Likewise, carbon dioxide concentration can be calculated from the following:

$$C_{CO_2} = \alpha'(P_{CO_2})(1 + \exp_{10}(pH - pK'))$$
 (28)

where, as above, pK' andα' are appropriate for temperature and pH. For 37°C:

$$C_{CO_2} = 0.0306(P_{CO_2})(1 + \exp_{10}(1.04214(pH) - 6.4104))$$
 (29)

DISCUSSION

The accuracy with which S_{O2} can be estimated from P_{O2} using an algorithm depends not only on how faithfully the algorithm represents the standard OHEC, but also on how "standard" the blood in question is. Shape and position of the OHEC can be quite nonstandard when there is carboxyhemoglobin, methemoglobin, or an abnormal hemoglobin present (Darling and Roughton, 1942; Roughton, 1964 and 1970; Enoki et al., 1969; Stamatoyannopoulos et al., 1971).

If 5 percent of the hemoglobin is liganded with carbon monoxide, as is common among urban cigarette smokers (Stewart et al., 1973), P_{50} of an otherwise standard OHEC is reduced from 26.6 to 25.1 torr, and the curve becomes more hyperbolic and less sigmoid. For such blood, S_{02} estimated from P_{02} according to the standard OHEC can be as much as 5.0 percent (saturation) lower than the true saturation of available hemoglobin (that not combined with carbon monoxide). For example, if P_{02} is 17.7 torr and carbon monoxide saturates 5 percent of the total hemoglobin, oxygen will actually saturate 31.0 percent of the remaining hemoglobin, whereas S_{02} estimated from the standard OHEC will be 26.0 percent.

Likewise, the presence of methemoglobin increases the affinity between oxygen and hemoglobin and produces a more hyperbolic OHEC (Darling and Roughton, 1942; Enoki et al., 1969). P₅₀ of blood containing abnormal hemoglobins can be as low as 4.6 torr (hemoglobin Hiroshima) or as high as 70 torr (hemoglobin Kansas), and shapes of the OHEC's for abnormal hemoglobins are often quite different from the shape of the standard curve (Stamato-yannopoulas et al., 1971). Furthermore, position of the OHEC is strongly dependent upon the intraerythrocyte concentrations of enzymes (hexokinase, pyruvate kinase) and substrates (2,3-diphosphoglycerate (2,3-DPG), adenosine

triphosphate), which can vary substantially in many pathologic and physiologic conditions (Thomas et al., 1974). For all these reasons, S_{0_2} estimated from P_{0_2} using the standard OHEC should be interpreted with caution.

Some would estimate even beyond S_{O_2} to oxygen concentration (C_{O_2}) . C_{O_2} can be calculated from a combination of measured P_{O_2} , measured hemoglobin concentration (Hb), estimated S_{O_2} (from virtual P_{O_2} , as explained in section IB), assumed solubility of oxygen in blood (α) (Hedley-Whyte and Laver, 1964), and assumed oxygen-carrying capacity of hemoglobin (β) (Dijkhuizen et al., 1977):

 $C_{O_2} = \alpha(P_{O_2}) + \beta(S_{O_2})$ (Hb) (30)

(The following values are conventionally used: $\alpha = 0.003 \text{ mlO}_2/(100 \text{ ml blood x})$ torr) and $\beta = 1.34-1.36 \text{ mlO}_2/\text{gmHb.})$ Besides all the potential errors associated with estimating S_{O_2} from P_{O_2} , uncertainty regarding the functional value of β for a given blood sample (Theye, 1970; Dijkhuizen et al., 1977) makes the estimated C_{O_2} tenuous, at best.

Before calculating virtual P_{O_2} from equation 5 or 5' when estimating S_{O_2} from P_{O_2} , values for pH, P_{CO_2} , and P_{O_2} should be corrected to body temperature with equations 8, 10, and 16, 17, 19, or 20, respectively. Contrary to the recommendation of some (Ruiz et al., 1975), it is incorrect to calculate virtual P_{O_2} from values of pH, P_{CO_2} , and P_{O_2} as measured at 37° C, using an equation that does not adjust for abnormal body temperature. This approach falsely assumes that physically warming or cooling the blood from body temperature to 37° C in a blood-gas analyzer results in the same change in P_{O_2} that would be expected from shift in position of the OHEC. As explained in section IIC, S_{O_2} actually increases when blood is cooled and decreases when blood is warmed, owing to transfer of oxygen between hemoglobin and the dissolved state. Therefore, when temperature is adjusted to 37° C for

measurement, P_{O_2} is incremented or decremented in such a way that the value of P_{O_2} at 37°C cannot be directly transformed, through a representation of the OHEC, to S_{O_2} at body temperature. Rather, the P_{O_2} at body temperature must first be estimated using an algorithm, such as equation 16, 17, 19, or 20, that corrects for change in S_{O_2} as well as for shift in position of the OHEC with warming or cooling. The value of P_{O_2} at body temperature could then be substituted in an equation defining the OHEC under in vivo conditions of temperature, pH, and P_{CO_2} . In practice, however, it is more convenient to calculate, with equation 5 or 5', a virtual P_{O_2} expected under standard conditions and then to compute S_{O_2} from an algorithm representing the OHEC at 37° C, pH = 7.40, and P_{CO_2} = 40 torr (table 2).

In contrast, values for pH and P_{CO_2} as measured at $37^{\circ}C$ can be directly substituted in equations 22-25 when estimating BE, BE₃, and BE_C. Base excess is defined in terms of relationships among pH, P_{CO_2} , and hemoglobin concentration in fully oxygenated blood at $38^{\circ}C$ (Astrup et al., 1960; Siggaard Andersen, 1962 and 1963). However, BE remains constant during anaerobic temperature change, because, despite thermally-induced alterations in pH and P_{CO_2} , the concentration of nonvolatile acid does not change with warming or cooling (Severinghaus et al., 1956). Therefore, when estimating BE, it should be permissable to use values of pH and P_{CO_2} at $37^{\circ}C$ or at body temperature. For reasons both practical and theoretical, it is preferable to use values as measured at $37^{\circ}C$, near the temperature at which titrations were carried out for experimental definition of BE in terms of pH, P_{CO_2} , and hemoglobin concentration (Astrup et al., 1960; Siggaard Andersen, 1962 and 1963).

Algorithms for calculating P_{O_2} from S_{O_2} can be used to estimate the P_{50} of a blood sample from one set of measurements of P_{O_2} , pH, P_{CO_2} , and S_{O_2} . To

do this, measured P_{O_2} is standardized to 37° C, pH = 7.40, and P_{CO_2} = 40 torr, using an equation for calculating virtual P_{O_2} :

$$P_{s} = P_{m}(\exp_{10}(-0.024(T-37)+0.40(pH-7.4)+$$

$$0.06(\log_{10}^{40}-\log_{10}^{40}P_{CO_{2}})))$$
(31)

where P_s is the standardized and P_m the measured P_{O_2} . As when calculating P'_v with equation 5', independent variables should be corrected to body temperature before substitution in equation 31. The P_{O_2} that would be expected (P_e) from the measured S_{O_2} , if the blood under study had standard oxygen-hemoglobin equilibrium, is calculated using one of the algorithms in table 3. The ratio of observed-to-expected P_{O_2} (P_s/P_e) provides a factor by which all P_{O_2} is in the midportion of the OHEC are shifted to the right or left of the standard curve. Therefore, multiplying P_{50} of the standard OHEC (26.6 torr) by this ratio gives an estimate of the P_{50} of the blood under standard conditions of temperature, pH, and P_{CO_2} (Weiskopf and Severinghaus, 1972; Aberman et al., 1975).

Aberman and colleagues found good reproducibility when using this technique to determine the P_{50} of 92 blood samples having S_{O_2} between 20 and 90 percent; for 3-6 replications on each sample, standard deviation of P_{50} was ± 1.0 torr. They also found good agreement between the "one-point" technique and the more traditional method of plotting three or more values of S_{O_2} against corresponding P_{O_2} , with or without Hill's transformation (1910)⁵, and reading P_{50} from the curve that best fits the points (Aberman et al., 1975).

Of more physiologic relevance than the standardized P_{50} described above is the P_{50} of blood under in vivo conditions of temperature, pH, and P_{CO_2} . To estimate the in vivo P_{50} , only P_{O_2} and S_{O_2} need to be measured. Because the one-point technique for estimating P_{50} is more accurate when S_{O_2} is between 20 and 90 percent (Aberman et al., 1975), S_{O_2} of the blood sample should be

tonometerd to, if not already in, that range. Measured P_{O_2} is corrected to body temperature using equations 16, 17, 19, or 20 to give the observed P_{O_2} (P_o), while expected P_{O_2} (P_e) is calculated from measured P_{O_2} with an algorithm from table 3. Then, just as for the standardized P_{S_0} , in vivo P_{S_0} is calculated by multiplying 26.6 torr by the ratio of observed-to-expected P_{O_2} (P_o/P_e). By reflecting influences of body temperature and blood pH and P_{CO_2} on the affinity between oxygen and hemoglobin, the in vivo P_{S_0} provides a more realistic appraisal of the availability of oxygen to body tissues than does the standardized P_{S_0} .

It is becoming increasingly more evident that relationships among P_{O_2} , S_{O_2} , pH, P_{CO_2} , Hb, 2,3-DPG, and temperature are more complex than previously thought. For example, not only is $\Delta \log_{10} P_{O_2}/\Delta T$ reduced progressively as S_{O_2} is increased above 75-80 percent (fig. 1a), but $\Delta \log_{10} P_{O_2}/\Delta T$ is reduced further, when S_{O_2} is above 90 percent, by hypercapnia and by anemia (Kelman, 1968). In blood with a low concentration of 2,3-DPG, $\Delta \log_{10} P_{O_2}/\Delta PH$ is lower than normal when pH is changed with nonvolatile acid, but higher than normal when pH is changed with carbon dioxide (Wranne et al., 1972). Whether the concentration of 2,3-DPG is low or normal, $\Delta \log_{10} P_{O_2}/\Delta PH$ owing to change in P_{CO_2} is greater at lower than at higher S_{O_2} (Hlastala and Woodson, 1975). When BE is substantially reduced (for example, to -10 mEq/L), $\Delta \log_{10} P_{O_2}/\Delta PH$ is additionally increased at all saturations (Adams et al.,1968). These interactions, and others like them, may require consideration in physiologic investigations, but they probably don't have to be accounted for in algorithms used for clinical purposes.

Kelman's (1966) and Thomas' (1972) algorithms for estimating S_{O_2} from P_{O_2} involve the same polynomial equation:

$$S = 100(a_1 P + a_2 P^2 + a_3 P^3 + P^4)/(a_4 + a_5 P + a_6 P^2 + a_7 P^3 + P^4)$$
 (32)

Using the following variation when programming for small calculators can result in calculation times that are one-third to one-half as long as when the above expression is straightforwardly programmed:

$$S = 100((P(a_1+P(a_2+P(a_3+P)))))/(a_4+P(a_5+P(a_6+P(a_7+P))))$$
(33)

Whereas it takes only 2 sec to solve the specially—actored version of the Kelman-Thomas equation on the HP-67 programmable calculator, the conventional form takes about 4 sec when powers are generated by sequential multiplication and about 6 sec when the y^X function is used. Computation times in table 4 for polynomial equations of Thomas, Adair, Kelman, and Tien reflect this programming strategy.

While calculating the P_{O_2} at body temperature from that measured at 37°C, it is convenient to correct for a common measuring artifact. Because of a diffusion gradient for oxygen through liquid in a conventional oxygen analyzer, the measured P_{O_2} is lower than the true blood P_{O_2} (Polgar and Furster, 1960; Moran et al., 1966; Rhodes and Moser, 1966; Adams et al., 1968; Severinghaus, 1968; Hulands et al., 1970; Severinghaus and Bradley, 1971). This difference would not be seen if the blood were continually stirred during analysis. Blood P_{O_2} is usually underestimated by about 4 percent when currently-available commercial electrodes are calibrated directly with gas. This error can be corrected with a factor determined by measuring the P_{O_2} in air and in water tonometered with air at 37°C. To account for slower diffusion of oxygen through blood than through water, the difference between P_{O_2} in air and in water is multiplied by 1.8. The difference is then divided by the P_{O_2} in air to give a factor with which to correct blood P_{O_2} readings for "stirring artifact":

$$P_{c} = P_{m}(1 + (1.8(P_{a} - P_{w})/P_{a}))$$
ected $P_{O_{a}}$ and measure $P_{O_{a}}$

where P_c and P_m are corrected P_{O_2} and measured P_{O_2} , respectively, and P_a and P_w are P_{O2} values in air and in water tonometered with air at 37°C (Severing-(34)haus and Bradley, 1971). If the oxygen electrode is calibrated with phometered water, a different correction for stirring artifact is required (Hylands et al., 1970).

When the liquid junction of the pH electrode is at 37°C, there is also an artifact in the measurement of blood pH. Owing to greater attraction of erythrocytes for potassium ions than for chloride ions at the liquid junction, the "suspension effect" results in an apparent pH of whole blood that is approximately 0.01 lower than that of true plasma (Jenny et al., 1950; Severinghaus et al., 1956; Mattock, 1959; Siggaard Andersen, 1961; Severinghaus and Bradley, 1971). Therefore, accuracy of blood pH measurements may be improved by adding 0.01 to measured values. Alternatively, a specific correction factor for a given electrode can be determined by measuring pH differences between whole blood and true plasma in multiple samples (Severinghaus et al., 1956). Correction is not required if the liquid junction is at room temperature, because then an equal and opposite potential difference automatically corrects for the suspension effect (Severinghaus and Bradley, 1971).

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The views, opinions, and/or findings in this report are those of the author and should not be construed as an official Department of Army position, policy, or decision, unless so designated by other official documentation.

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CAPTIONS FOR FIGURES

- Fig. 1. Factors to correct P_{O_2} for anaerobic temperature change are dependent upon P_{O_2} .
 - a. Values of $\Delta \log_{10} P_{O_2}/\Delta T$ from Kelman and Nunn's algorithm are as much as 27 percent lower than those reported by Severinghaus in 1966. When S_{O_2} is 99 percent or less, values of $\Delta \log_{10} P_{O_2}/\Delta T$ calculated from Ruiz's or Hewlett-Packard's algorithm fall within 2 percent of the Severinghaus graph.
 - b. To correct P_{O_2} from 37°C to body temperature, the measured value of P_{O_2} is multiplied by the calculated ratio of P_{O_2} at body temperature (P_T) to that at 37°C (P_{37}). P_T/P_{37} calculated from Ruiz's or Hewlett-Packard's algorithm differ, for any P_{O_2} , with values calculated from the Severinghaus graph by less than 0.4 percent per degree centigrade.
 - Values for P_T/P_{37} calculated from the Kelman-Nunn algorithm differ by as much as 1.3 percent per degree centigrade with those calculated from the graph of Severinghaus (or from the algorithms of Ruiz and Hewlett-Packard). Percentage differences are maximum at $P_{02} = 110-130$ torr. Kelman and Nunn's corrections for temperature are lower than those of Severinghaus when P_{02} is greater than about 65 torr and are higher than those of Severinghaus elsewhere (fig. 1a).

When correcting a value of P_{O_2} greater than 65 torr for hypothermia, the ratio P_T/P_{37} calculated from the Kelman-Nunn algorithm is larger than that calculated from the Severinghaus data; that is, P_T/P_{37} is closer to unity, which represents a lesser correction (fig. 1b).

- Fig. 2. To calculate BE using Gershwin's algorithm, slope of the blood buffer line is approximated by substituting a value for Hb and an estimate for BE in an empirically-derived equation (23a). Equations representing the blood buffer line (rectilinear) and the base excess curve (hyperbolic), each expressed in terms of pH and $\log_{10}P_{\text{CO}_2}$, are simultaneously solved for pH, eliminating the P_{CO_2} terms. The resulting value, pH', is the abscissa of the point of intersection between the blood buffer and base excess lines. This value is then substituted in equation 23c expressing BE in terms of pH alone. Equation 23c mathematically represents reflection of the base excess curve on the horizontal axis. Respective values for BE and pH, according to this equation, can be read from points at which vertical lines intersect the base excess curve and the horizontal axis.
- Fig. 3. When P_{CO_2} is high and hemoglobin concentration is low, Gershwin's algorithm overestimates values of BE to a progressively greater degree as BE and P_{CO_2} increase and as Hb decreases.

TABLE 1

Data Defining Severinghaus' Standard Oxygen Hemoglobin

Equilibrium Curve (OHEC)

S _{O2}	PO2	S _{O2}	P _{O2}
1	1.9	85	49.8
2	3.4	90	57.8
4	5.7	91	60.0
6	7.5	92	62.7
10	10.3	93	65.7
15	13.1	94	69.4
20	15.4	95	74.2
25	17.3	95.5	77.3
30	19.2	96	81.0
35	21.0	96.5	86.0
40	22.8	97	91.6
45	24.6	97.5	99.6
50	26.6	98	111.
55	28.7	98.5	129.
60	31.2	99	159.
65	34.0	99.5	225.
70	36.9	99.8	350.
75	40.4	99.9	500.
80	44.5	99.95	700.

Algorithms for Calculating So (S) from Po (P) According to the Standard Oxygen Hemoglogbin Equilibrium Curve

Ruiz et al. (1975):	Aberman et al. (1973):	Lutz et al. (1975):
$S = 99.95 - 100/(1 + ((P+7)/33.7)^{3.3})$	$S = \sum_{i=1}^{i=7} a_{i+1} ((P-27.5)/(P+27.5))^{i}$	For S = 1-40%:
-0.5/(1+((P-130)/35) ²)+0.45/(1+((P-68)/12) ⁶)	i=0 i=i	$S = a_1 P^2 + a_2 P$
-0.5/(1+((P-35)/3) ⁴)-0.5/(1+((P-15)/4) ⁴)	$a_1 = 5.187074 \times 10^1$	For S = 40-56%:
+0.35/(1+((P-26)/3) ⁶)+0.2/(1+((P-53)/8) ⁴)	$a_2 = 1.298325 \times 10^2$	$S = a_3 P - a_4$
$-0.4/(1+((P-40)/0.9)^4)-0.2/(1+((P-200/65)^8)$	$a_3 \approx 6.828367 \times 10^0$	For S = 56-100%;
$+0.4/(1+((P-9)/3)^2)$	$a_{\mu} = -2.237881 \times 10^2$	$S = 100/((a_5/P)^6 + 1)$
	$a_5 \approx -2.795300 \times 10^1$	a ₁ = 0.062
	$a_6 = 2.585009 \times 10^2$	$a_2 = 0.351$
	$a_7 = 2.184175 \times 10^1$	$a_3 = 2.50$
	$a_8 = 1.192322 \times 10^2$	$a_{4} = 16.9$
•	•	$a_5 = 27.0$
		a ₆ = 2.80

Thomas (1972): $S = 100(a_1 P + a_2 P^2 + a_3 P^3 + P^4)/(a_4 + a_5 P + a_6 P^2 + a_7 P^3 + P^4)$ $a_1 = -2.000 \times 10^3$ $a_2 = 2.045 \times 10^3$ $a_3 = 1.500 \times 10^4$ $a_4 = 2.400 \times 10^4$ $a_5 = 3.110 \times 10^4$ $a_6 = 2.400 \times 10^3$ $a_7 = 1.500 \times 10^1$

 $a_2 = 7.80 \times 10^{-4}$ $a_3 = 4.44 \times 10^{-6}$

 $a_{\mu} = 2.55 \times 10^{-6}$

 $S = 100(a_1^{P+2}a_2^{P^2+3}a_3^{P^3+4}a_4^{P^4})/(4(1+a_1^{P+}a_2^{P^2}+a_3^{P^3}+a_4^{P^4}))$ $a_1 = 2.57 \times 10^{-2}$

Adair (Roughton-Severinghaus (1973)):

Severinghaus #1 (1979): $_7P^3+P^4$) $S = 100/(1+23,400/(P(P^2+150)))$	Severinghaus #2: S = 100/(1+7070/P ^{2.7})	Severinghaus #3 (1976b): For S = 35-100%: S = 100-189exp _e (-0.05(P))
Kelman (1966): $S = 100(a_1 P + a_2 P^2 + a_3 P^3 + P^4)/(a_4 + a_5 P + a_6 P^2 + a_7 P^3 + P^4)$	$a_1 = -6.5522285 \times 10$ $a_2 = 2.1214010 \times 10^3$ $a_3 = -6.7073989 \times 10^1$	$a_{4} = 3.5996087 \times 10$ $a_{5} = -3.1346258 \times 10^{4}$ $a_{6} = 2.3961674 \times 10^{3}$ $a_{7} = -6.7104406 \times 10^{1}$

*Personal communication: John W. Severinghaus, M.D., Department of Anesthesia, School of Medicine, University of California, San Francisco, CA 04112

TABLE 3

Algorithms for Calculating Po2 (P) from So2 (S) According to the Standard Oxygen Hemoglobin Equilibrium Curve

Tien et al. (1977):		Lutz et al. (1975);
$P = \exp(a_0 + a_1 x + a_2 x^2 + + a_n x^n)$	+a,x")	For S = 1-40%:
where x is the natur	where x is the natural logarithm of S/(100-S)	$P = (((a_2/2)^2 + a_1 S)^{0.5} - a_2/2)/a_1$
For S = 30-91%:	$a_0 = 3.280 \times 10^0$	For S = 40-56%:
	$a_1 = 3.910 \times 10^{-1}$	$P = (S + a_{\mu})/a_{3}$
	$a_2 = 2.124 \times 10^{-2}$	For S = 56-98%:
	$a_1 = -2.000 \times 10^{-2}$	$P = a_{\varsigma}(S/(100-S))^{1/a}6$
	$a_h = -2.332 \times 10^{-2}$	a ₁ = 0.062
	$a_5 = 1.898 \times 10^{-2}$	$a_2 = 0.351$
	$a_c = -3.564 \times 10^{-3}$	a ₃ = 2.50
For S = 30-96%:	$a_0 = 3.2801 \times 10^0$	$a_{\mu} = 16.9$
	$a_1 = 3.9246 \times 10^{-1}$	$a_5 = 27.0$
	$a_2 = 2.1870 \times 10^{-2}$	a _k = 2.80
	$a_1 = -2.6553 \times 10^{-2}$,
	$a_{\mu} = -2.1251 \times 10^{-2}$	
*	$a_5 = 2.4607 \times 10^{-2}$	
	$a_{k} = -7.8654 \times 10^{-3}$	
	$a_7 = 8.5292 \times 10^{-4}$	Severinghaus (1979):
For S = 20-97.5%:	$a_0 = 3.283 \times 10^0$	$P = \exp_{(0.385\log_{e}(S/(100-S))+}$
	$a_1 = 3.909 \times 10^{-1}$	3.32-(0.72(5)) ⁻¹ -5 ⁶ /6(10 ¹²))
	$a_2 = -6.580 \times 10^{-4}$	

 $a_{\mu} = -3.805 \times 10^{-3}$ $a_{5} = 2.856 \times 10^{-3}$

 $a_2 = -6.580 \times 10^{-4}$ $a_3 = -9.348 \times 10^{-3}$ a6 = -3.494 x 10-4

Deviations from the Standard Curve and Computation Times for Algorithms Describing the Oxygen Hemoglobin Equilibrium Curve TABLE 4

	So ₂ from Po ₂	c	Range Erre (percer	ange of Resid Errors in S _O rcent saturat	ual ion)	Root Mean Square of Residual Errors in So (percent saturation) ²	Time (sec) ** to Solve Algorithm with Typical Small Programmable Calculator	
-	Ruiz et al. (1975)	38	-0.15 to	5	0.19	0.07	20.0	
5.	Aberman et al. (1973)	38	-0.36 to	9	0.40	0.17	13.5	
3.	Severinghaus #1 (1979)	38	-0.74	ţ.	64.0	0.34	1.5	
	Lutz et al. (1975)	38	99.0-	ţ	09.0	0.37		
	S _{O.} = 1-40%	01	(-0.41	9	0.24)****	(0.23)***	1.0	
	S _O = 40-56%	4	04.0-)	ţ	0.10)	(0.30)	0.5	
	$S_{0}^{-2} = 56-100\%$	54	99.0-)	\$	09.0	(0.42)	2.0	
	Thomas (1972)	38	-0.76	\$	92.0	0.38	2.0	
	Adair(Roughton-Severinghaus(1973))	38	-0.83	ţ.	1.19	0.57	2.5	
	Severinghaus #2*	38	-2.84	9	0.97	1.08	2.0	
	Kelman (1966)	38	-2.92	5	1.98	1.07	2.0	
	Severinghaus #3 (1976b)							
	S _{O2} = 35-99.95%	53	(-1.14 to	\$	1.27)	(0.63)	1.5	

Table 4 (Continued)

Po ₂ from So ₂	c	Range	e of Resi ors in P _C (torr)	Range of Residual Errors in P _{O2} (torr)	Root Mean Square of Residual Errors in P _{O2} (torr)	Time (sec)** to Solve Algorithm with Typical Small Programmable Calculator
I. Tien et al. (1977)						
S _{O2} = 30-91%	14	(-0.10	9	0.06)	(0.04)	5.0
So_ = 30-96%	20	(-0.09	\$	0.07)	(0.04)	5.5
$S_{O_2}^2 = 20-97.5\%$	25	(-0.21	\$	0.16)	(0.09)	5.0
2. Lutz ét al. (1975)						
So ₂ = 1-40%	10	(-0.12	\$	0.18)	(0.12)	2.0
SO2 = 40-56%	4	(-0.04	\$	0.16)	(0.12)	1.5
So = 56-98%	18	(-2.61	\$	3.09)	(1.90)	2.5
3. Severinghaus (1979)						
S _{O2} = 1.96%	28	(-0.72	9	0.72)	(0.33)	0.4
$S_{O_2}^2 = 1-97\%$	30	(-1.12	\$	0.72)	(0.39)	4.0

* Personal communication: John W. Severinghaus, M.D.

** To the nearest half second

*** Hewlett-Packard HP-67

**** Values for less than the full OHEC are in parentheses.

TABLE 5
Ranges of Base Excess (BE and BE₃), in mEq/L, Calculated from Algorithms within the Ranges: $P_{CO_2} = 20-80$ Torr, pH = 7.2-7.6

BE or BE ₃ from Nomogram or Blood Gas Calculator	BE from Siggaard Andersen's Algorithm*	BE from Gershwin's Algorithm*	BE ₃ from Severinghaus' Algorithm **
-15	-13.1 to -14.4	-14.6 to -15.1	-15.2 to -15.2
	(+1.9 to +0.6)***	(+0.4 to -0.1)	(-0.2 to -0.2)
-10	-8.3 to -9.5	-9.8 to -10.1	-10.1 to -10.2
	(+1.7 to +0.5)	(+0.2 to -0.1)	(-0.1 to -0.2)
-5	-3.7 to -5.0 (+1.3 to 0.0)	-4.9 to -5.6 (+0.1 to -0.6)	-5.1 to -5.3 (-0.1 to -0.3)
0	+0.1 to +1.4	-0.7 to +0.4	-0.1 to 0.0
	(+0.1 to +1.4)	(-0.7 to +0.4)	(-0.1 to 0.0)
+5	+5.0 to +6.2	+4.7 to +5.7	+4.8 to +5.1
	(0.0 to +1.2)	(-0.3 to +0.7)	(-0.2 to +0.1)
+10	+9.9 to +10.8	+9.5 to +10.8	+10.0 to +10.3
	(-0.1 to +0.8)	(-0.5 to +0.8)	(0.0 to +0.3)
+15	+15.3 to +16.3	+14.8 to +17.4	+14.8 to +15.3
	(+0.3 to +1.3)	(-0.2 to +2.4)	(-0.2 to +0.3)

^{*} Compared with the Blood Gas Calculator in the range of Hb = 10-20 gm/100ml

^{**} Compared with Siggaard Andersen's alignment nomogram, Hb = 3 gm/100ml

^{***} In parentheses are the ranges of differences between values calculated from the algorithms and those read from the nomogram or the Blood Gas Calculator.

FOOTNOTES

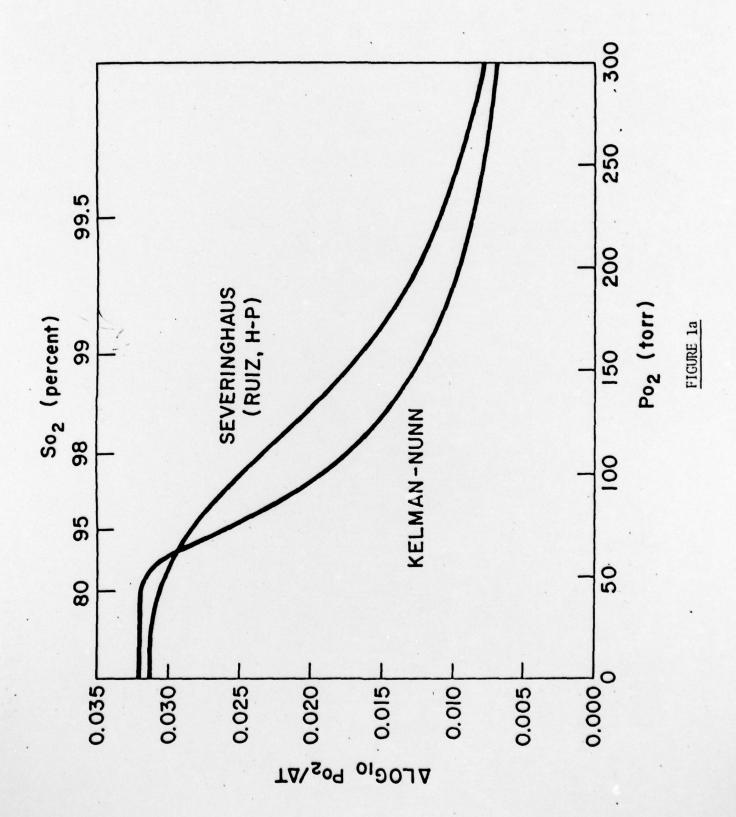
Supported in part by Grant GM 15904 from the National Institutes of Health,
United States Public Health Service

²The BGC, a sliderule having one degree of freedom more than a nomogram, was designed by Severinghaus, in collaboration with Siggaard Andersen, to represent the original BE titration data better than could Siggaard Andersen's alignment nomogram. (Personal communication: John W. Severinghaus, M.D.) Therefore, the BGC constitutes a higher standard for comparison than the alignment nomogram.

 3 BE = -13.1 from the algorithm compared with -15.0 mEq/L from the BGC when $P_{CO_{2}} = 20$ torr, pH = 7.302, and Hb = 20 gm/100ml.

 4 BE = +16.3 from the algorithm compared with +15.0 mEq/L from the BGC when 9 CO₂ = 39.2 torr, pH = 7.60, and Hb = 15 gm/100ml.

Sigmoidicity of the OHEC is transformed to near rectilinearity by plotting $\log_{10}(S_{O_2}/(100-S_{O_2}))$ against $\log_{10}P_{O_2}$.



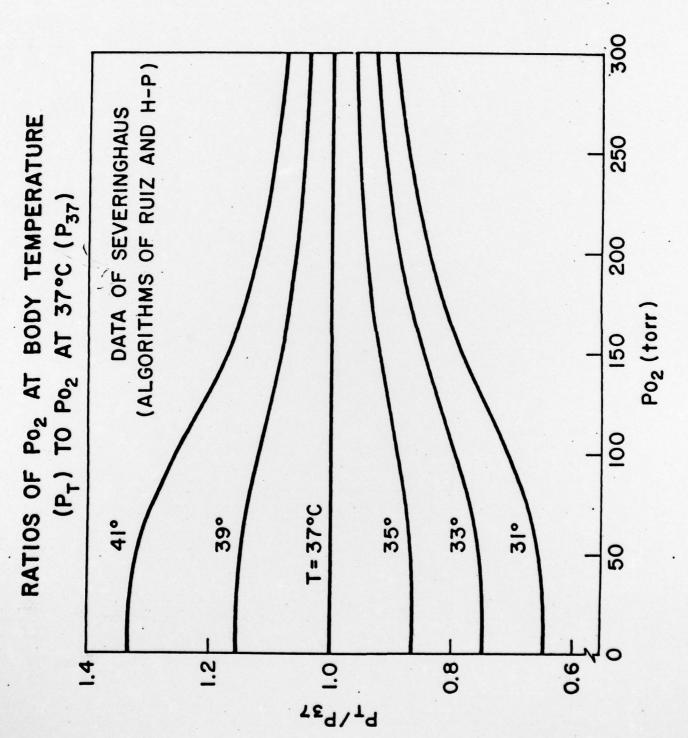


FIGURE 16

KELMAN AND NUNN'S ALGORITHM COMPARED WITH

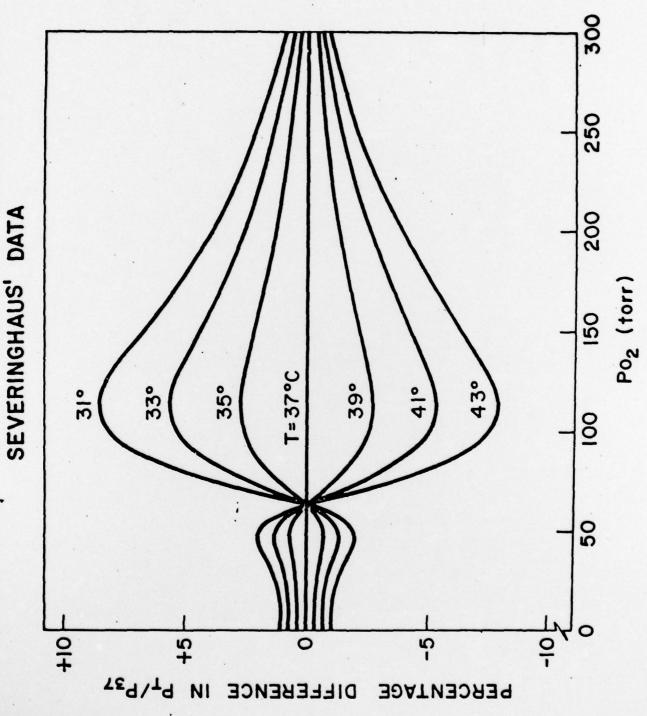


FIGURE 1c

SIGGAARD ANDERSEN'S ACID BASE NOMOGRAM

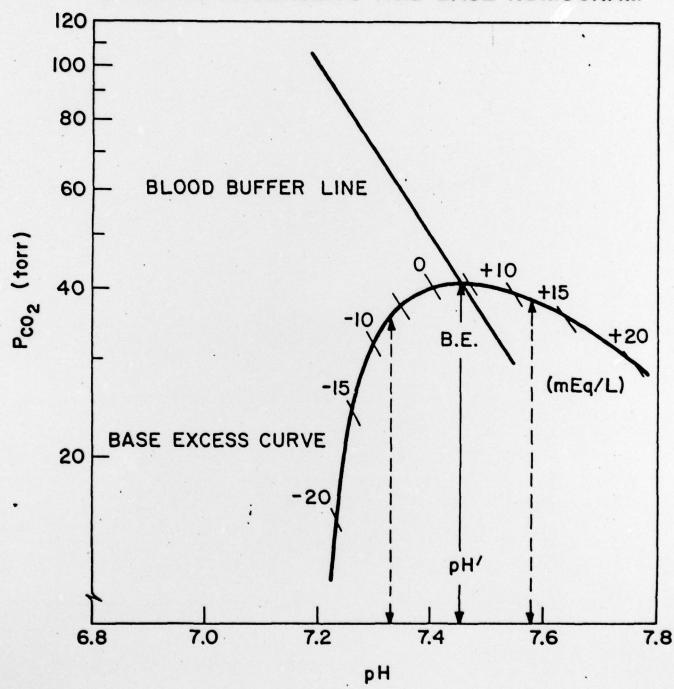


FIGURE 2

OVERESTIMATION OF BASE EXCESS BY GERSHWIN'S ALGORITHM

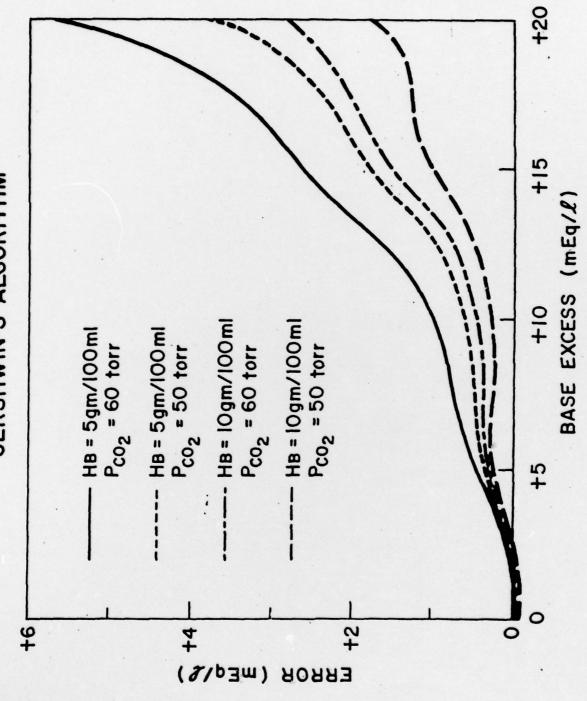


FIGURE 3